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The photodisintegration of helium

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Abstract. We calculate the integrated and the bremsstrahlung-weighted cross sections in the photodisintegration of ^{4}He by applying the sum rules of Levinger and Bethe and using the velocity-dependent potential of Nestor *et al.* The wave function used is a mixture of the $^1S_0$ and the principal $^5D_0$ states, the radial dependence of both these states being Gaussian. We obtain the parameters of the wave function from a variational calculation of the binding energy. The results show good agreement with experiments and with similar calculations using hard-core potentials.

1. Introduction

The study of the integrated cross section ($\sigma_{\mathrm{int}} = \int \sigma(w)\, \mathrm{d}w$) and bremsstrahlung-weighted cross section ($\sigma_{\mathrm{b}} = \int (\sigma/w)\, \mathrm{d}w$) in the photodisintegration of light nuclei is important from the point of view of nuclear forces and the ground-state wave function of these nuclei. In the present article we report such a calculation for the alpha particle.

Earlier, Rustgi and Levinger (1957), Srivastava and Jain (1967), and Lim (1968) have calculated these cross sections for the alpha particle by applying the sum rules of Levinger and Bethe (1950). However, these calculations are not realistic in the sense that either they do not include velocity-dependent (or hard-core) forces (Rustgi and Levinger 1957) or they do not contain a tensor component (Srivastava and Jain 1967 and Lim 1968). Goldhammer and Valk (1962) have used hard-core potentials in their calculations, but the value they obtain for the D-state probability is rather too large. In the present investigation we use the velocity-dependent potential of Nestor *et al.* (1968) which contains a static tensor component and evaluate the integrated and the bremsstrahlung-weighted cross sections for the alpha particle by applying the sum rules of Levinger and Bethe (1950). We compare our results for these cross sections with the experimental measurements of Gorbunov and Spiridonov (1957, 1958) to examine to what extent our model (ground-state wave function and interactions) for the alpha particle is satisfactory. We also compare our values of σ_{int} and σ_{b} with those of Goldhammer and Valk (1962) to study the equivalence of hard-core and velocity-dependent potentials in photo-effect calculations.

2. The potential and the ground-state wave function

The form of the potential of Nestor *et al.* (1968) is given by

$$U_j = \frac{\hbar^2}{m}\left[V_j^{\mathrm{C}}(r) + \left\{\left(\frac{p^2}{\hbar^2}\right)\omega_j(r) + \omega_j(r)\left(\frac{p^2}{\hbar^2}\right)\right\} + V_j^{\mathrm{T}}(r)S_{12} + V_j^{\mathrm{LS}}(r)\boldsymbol{L}\,.\,\boldsymbol{S}\right] \qquad (1)$$

where

$$V_j(r) = -A_j \exp\left(-\frac{1}{2}\frac{r^2}{\alpha_j^2}\right) \qquad (2)$$

$$\omega_j(r) = B_j \exp\left(-\frac{1}{2}\frac{r^2}{\beta_j^2}\right) \qquad (3)$$

for all potentials except the tensor part $V_j^{\mathrm{T}}(r)$, for which

$$V_j^{\mathrm{T}}(r) = -A_j\left(\frac{1}{2}\frac{r^2}{\alpha_j^2}\right)\exp\left(-\frac{1}{2}\frac{r^2}{\alpha_j^2}\right). \tag{4}$$

The index j stands for the four parts of the interaction: singlet–even, singlet–odd, triplet–even and triplet–odd.

We use a wave function which is symmetric in the spatial coordinates of all the four nucleons and therefore do not specify the potentials in the odd states. In our calculation we use the set 'B' of parameters in the potential of Nestor *et al.* (1968). The values of the potential parameters in the singlet–even state are

$$A_{\mathrm{C}}{}^{\mathrm{s}} = 2{\cdot}269\,\mathrm{fm}^{-2}, \quad \alpha_{\mathrm{C}}{}^{\mathrm{s}} = 0{\cdot}877\,\mathrm{fm}, \quad B^{\mathrm{s}} = 0{\cdot}6, \quad \beta^{\mathrm{s}} = 0{\cdot}877\,\mathrm{fm},$$
$$A_{\mathrm{LS}}{}^{\mathrm{s}} = 0 \quad \text{and} \quad \alpha_{\mathrm{LS}}{}^{\mathrm{s}} = 0. \tag{4a}$$

In the triplet–even state these are

$$A_{\mathrm{C}}{}^{\mathrm{t}} = 6{\cdot}825\,\mathrm{fm}^{-2}, \quad \alpha_{\mathrm{C}}{}^{\mathrm{t}} = 0{\cdot}598\,\mathrm{fm}, \quad B^{\mathrm{t}} = 1{\cdot}0, \quad \beta^{\mathrm{t}} = 0{\cdot}598\,\mathrm{fm},$$
$$A_{\mathrm{T}} = 0{\cdot}490\,\mathrm{fm}^{-2}, \quad \alpha_{\mathrm{T}} = 1{\cdot}20\,\mathrm{fm}, \quad A_{\mathrm{LS}}{}^{\mathrm{t}} = 0 \quad \text{and} \quad \alpha_{\mathrm{LS}}{}^{\mathrm{t}} = 0. \tag{4b}$$

The superscripts s and t denote the singlet and triplet states, respectively.

We assume the ground-state wave function of the alpha particle to be represented by a mixture of the $^1\mathrm{S}_0$ and the principal $^5\mathrm{D}_0$ states, so that the complete wave function may be written in the form (Irving 1953)

$$\psi = \frac{1}{(1+C^2)^{1/2}}(\psi_{\mathrm{S}} + C\psi_{\mathrm{D}}). \tag{5}$$

In the above equation, ψ_{S} and ψ_{D} are separately normalized to unity, so that ψ is normalized to unity and C^2 determines the amount of D-state in the mixture.

We choose the form of the radial wave functions to be Gaussian in the relative spatial coordinates of the nucleons (Gerjuoy and Schwinger 1942).

The transformation (Irving 1953)

$$\boldsymbol{v} = (\boldsymbol{r}_2 - \boldsymbol{r}_1)/\sqrt{2}, \quad \boldsymbol{w} = (\boldsymbol{r}_4 - \boldsymbol{r}_3)/\sqrt{2}, \quad \boldsymbol{u} = \tfrac{1}{2}(\boldsymbol{r}_4 + \boldsymbol{r}_3 - \boldsymbol{r}_2 - \boldsymbol{r}_1),$$
$$\boldsymbol{R} = \tfrac{1}{4}(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 + \boldsymbol{r}_4) \tag{6}$$

gives

$$\psi_{\mathrm{S}} = N_{\mathrm{S}} \exp\{-2\mu(u^2 + v^2 + w^2)\}\chi \tag{7}$$

and

$$\psi_{\mathrm{D}} = N_{\mathrm{D}} \exp\{-2\nu(u^2+v^2+w^2)\}\{6(\boldsymbol{\sigma}_1 \,.\, \boldsymbol{v})(\boldsymbol{\sigma}_3 \,.\, \boldsymbol{w}) + 6(\boldsymbol{\sigma}_1 \,.\, \boldsymbol{w})(\boldsymbol{\sigma}_3 \,.\, \boldsymbol{v})$$
$$- 4(\boldsymbol{\sigma}_1 \,.\, \boldsymbol{\sigma}_3)(\boldsymbol{v} \,.\, \boldsymbol{w})\}\chi. \tag{8}$$

The normalization constants in the new coordinate system are

$$N_{\mathrm{S}} = 2^{9/2}\mu^{9/4}/\pi^{9/4} \quad \text{and} \quad N_{\mathrm{D}} = 2^{11/2}\nu^{13/4}/3\sqrt{5}\pi^{9/4} \tag{9}$$

and the spin wave function is

$$\chi = \tfrac{1}{2}(\chi_1{}^+\chi_2{}^- - \chi_1{}^-\chi_2{}^+)(\chi_3{}^+\chi_4{}^- - \chi_3{}^-\chi_4{}^+) \tag{10}$$

where the subscripts 1, 2 denote the neutron and 3, 4 denote the proton coordinates.

A variational calculation of the binding energy of the alpha particle gives the best values of the parameters to be

$$\mu = 0{\cdot}17\,\mathrm{fm}^{-2}, \quad \nu = 0{\cdot}25\,\mathrm{fm}^{-2} \quad \text{and} \quad C = -0{\cdot}11. \tag{11}$$

3. The integrated electric dipole absorption cross section

In the electric dipole approximation, the integrated cross section is given by (Levinger and Bethe 1950)

$$\sigma_{\mathrm{int}} = \frac{2\pi^2 e^2 \hbar}{mc} \sum_n f_{0n} \tag{12}$$

where $\Sigma_n f_{0n}$ is the summed oscillator strength.

In equation (12)

$$\sum_n f_{0n} = \Big(\sum_n f_{0n}\Big)_{\mathrm{T}} + \Big(\sum_n f_{0n}\Big)_{\text{static central}} + \Big(\sum_n f_{0n}\Big)_{\text{vel dep central}} + \Big(\sum_n f_{0n}\Big)_{\text{static tensor}} \tag{13}$$

where

$$\Big(\sum_n f_{0n}\Big)_{\mathrm{T}}, \quad \Big(\sum_n f_{0n}\Big)_{\text{static central}}, \quad \Big(\sum_n f_{0n}\Big)_{\text{vel dep central}}$$

and

$$\Big(\sum_n f_{0n}\Big)_{\text{static tensor}}$$

are, respectively, the contributions of the kinetic energy, the static central part of the potential, the velocity-dependent part of the potential and the static tensor part of the potential. As shown by Rustgi and Levinger (1957), and Srivastava and Jain (1967), these are given by

$$\Big(\sum_n f_{0n}\Big)_{\mathrm{T}} = (NZ/A) = 1 \text{ for } {}^4\mathrm{He} \tag{14}$$

$$\Big(\sum_n f_{0n}\Big)_{\text{static central}} = -\frac{m}{3\hbar^2}(x+\tfrac{1}{2}y)\Big\langle \sum_k \sum_l V^{\mathrm{C}}_{\mathrm{static}}(r_{kl}) r^2_{kl} \Big\rangle_{00} \tag{15}$$

$$\Big(\sum_n f_{0n}\Big)_{\text{vel dep central}} = \{\langle \omega^{\mathrm{s}}(r_{23})\rangle_{00} + \langle \omega^{\mathrm{t}}(r_{23})\rangle_{00}\} \tag{16}$$

$$\Big(\sum_n f_{0n}\Big)_{\text{static tensor}} = -\frac{m}{3\hbar^2}(x'+\tfrac{1}{2}y')\Big\langle \sum_k \sum_l V^{\mathrm{T}}_{\mathrm{static}}(r_{kl}) r^2_{kl} \Big\rangle_{00} \tag{17}$$

where k denotes protons and l denotes neutrons; x and x' are respectively the fractions of static central and tensor potentials that have Majorana exchange character; y and y' represent fractions of the static central and tensor potentials that have Heisenberg exchange character. The velocity-dependent part is assumed to have a Wigner character.

Orthogonality of S- and D-states reduces equation (12) to

$$\sigma_{\mathrm{int}} = \frac{2\pi^2 e^2 \hbar}{mc}\Bigg[1 - \frac{m}{(1+C^2)3\hbar^2}\Bigg\{(x+\tfrac{1}{2}y)\Bigg(\int \psi_{\mathrm{S}}{}^* \sum_k \sum_l V^{\mathrm{C}}_{\mathrm{static}}(r_{kl}) r^2_{kl} \psi_{\mathrm{S}}\,\mathrm{d}\tau$$

$$+ C^2 \int \psi_{\mathrm{D}}{}^* \sum_k \sum_l V^{\mathrm{C}}_{\mathrm{static}}(r_{kl}) r^2_{kl} \psi_{\mathrm{D}}\,\mathrm{d}\tau\Bigg)$$

$$+ (x'+\tfrac{1}{2}y')\Bigg(\int \psi_{\mathrm{S}}{}^* \sum_k \sum_l V^{\mathrm{T}}_{\mathrm{static}}(r_{kl}) r^2_{kl}\, \psi_{\mathrm{S}}\,\mathrm{d}\tau$$

$$+2C\int\psi_{\rm S}^*\sum_k\sum_l V^{\rm T}_{\rm static}(r_{kl})r_{kl}^2\psi_{\rm D}\,{\rm d}\tau+C^2\int\psi_{\rm D}^*\sum_k\sum_l V^{\rm T}_{\rm static}(r_{kl})r_{kl}^2\psi_{\rm D}\,{\rm d}\tau\Big)\Big\}$$

$$+\frac{1}{1+C^2}\Big(\int\psi_{\rm S}^*\{\omega^{\rm s}(r_{23})+\omega^{\rm t}(r_{24})\}\psi_{\rm S}\,{\rm d}\tau+C^2\int\psi_{\rm D}^*\{\omega^{\rm s}(r_{23})+\omega^{\rm t}(r_{23})\}\psi_{\rm D}\,{\rm d}\tau\Big)\Big]. \tag{18}$$

We evaluate matrix elements in equation (18) using the transformations and spin matrix elements given by Irving (1953) and get

$$\sigma_{\rm int}=\frac{2\pi^2e^2\hbar}{mc}\{1+0{\cdot}79855(x+\tfrac{1}{2}y)+0{\cdot}01988(x'+\tfrac{1}{2}y')+0{\cdot}20626\}. \tag{19}$$

Assuming $x = x'$ and $y = y'$ we get

$$\sigma_{\rm int}=\frac{2\pi^2e^2\hbar}{mc}\{1{\cdot}20626+0{\cdot}81843(x+\tfrac{1}{2}y)\} \tag{20}$$

$$= 96{\cdot}6 \text{ MeV mb for } x+\tfrac{1}{2}y = 0{\cdot}5 \tag{21}$$

(Serber mixture)

$$= 111{\cdot}1 \text{ MeV mb for } x+\tfrac{1}{2}y = 0{\cdot}8 \tag{22}$$

(Rosenfeld mixture).

4. The bremsstrahlung-weighted cross section

Foldy (1957) has shown that for a nucleus whose ground-state wave function is completely symmetric in the space coordinates of all the nucleons, the bremsstrahlung-weighted cross section is related to the mean-square radius $\langle r^2\rangle_{00}$ through the expression

$$\sigma_{\rm b}=\left(\frac{4}{3}\pi^2\right)\left(\frac{e^2}{\hbar c}\right)\left(\frac{NZ}{A-1}\right)\langle r^2\rangle_{00} \tag{23}$$

where

$$\langle r^2\rangle_{00}=\frac{1}{Z}\Big\langle\sum_{\rm p}(\boldsymbol{r}_{\rm p}-\boldsymbol{R})^2\Big\rangle_{00}=R_{\rm C}{}^2-R_{\rm p}{}^2. \tag{24}$$

In equation (24) $\boldsymbol{r}_{\rm p}$ denotes the proton coordinates, $\boldsymbol{R}$ denotes centre-of-mass coordinates, and $R_{\rm C}{}^2$ and $R_{\rm p}{}^2$ are the mean-square radii of the charge distribution in the nucleus and the proton, respectively.

Using equation (6), we get

$$\langle r^2\rangle_{00}=\langle\tfrac{1}{4}u^2+\tfrac{1}{2}w^2\rangle_{00} \tag{25}$$

which on evaluation reduces to

$$\langle r^2\rangle_{00}=\frac{1}{(1+C^2)}\left(\frac{9}{32\mu}+\frac{13C^2}{32\nu}\right). \tag{26}$$

Using the best values of parameters, μ, ν and C, obtained from the variational calculation of the binding energy, we find

$$\langle r^2\rangle_{00}=1{\cdot}66\text{ fm}^2 \tag{27}$$

and

$$\langle r^2\rangle_{00}{}^{1/2}=1{\cdot}29\text{ fm} \tag{28}$$

while $\{\langle r^2\rangle_{00}{}^{1/2}\}_{\text{expt}} = 1{\cdot}44$ fm (from electron – helium scattering experiments). Thus our value of the root-mean-square radius of the alpha particle is in reasonable agreement with that obtained from electron–helium scattering experiments (Hofstadter 1956, Frosch *et al.* 1966). Finally, equations (23) and (27) give

$$\sigma_b = 2{\cdot}13\,\text{mbn}. \tag{29}$$

5. Discussion

Our results and the results of earlier theoretical calculations of σ_{int} and σ_b, and also the experimental values of Gorbunov and Spiridonov (1958), are shown in table 1.

Table 1. Values of the integrated and the bremsstrahlung-weighted cross sections for ^{4}He

Reference	σ_{int}(MeV mbn) Serber mixture	Rosenfeld or Inglis mixture	σ_b (mbn)
Present calculation	96·6	111·1	2·13
Rusti and Levinger[1]	88·8	106·5	0·80
Rustgi and Levinger[2]	86·3	102·0	1·23
Jain and Srivastava[3]	101·0	123·0	3·80
Goldhammer and Valk[4]	107·0	—	2·73
Experimental	95 ± 7		2·40 ± 0·15

[1] With a central static potential without a hard core (Rustgi and Levinger 1957).
[2] With a tensor static potential without a hard core (Rustgi and Levinger 1957).
[3] With a central velocity-dependent potential (Srivastava and Jain 1967).
[4] With a repulsive core and a tensor component (Goldhammer and Valk 1962).

We see that for purely central forces without hard core (Rustgi and Levinger 1957) the value of σ_b is very low. Although the tensor forces (Rustgi and Levinger 1957) improve the calculated value of σ_b, it still remains considerably lower than the experimental value. With a velocity-dependent central potential, Srivastava and Jain (1967) obtained values for both σ_b and σ_{int} much higher than the experimental values. In our present calculation, inclusion of the tensor forces in the velocity-dependent potential has reduced their values appreciably resulting in a better agreement with the experiments. We further note that the values of σ_b and σ_{int} for the Serber mixture calculated by Goldhammer and Valk (1962) are somewhat higher than our values and the experimental values. The considerably large values of Goldhammer and Valk (1962) may be attributed to the unusually large D-state probability of 10·6%. A comparison of our values of σ_{int} to those of Rustgi and Levinger (1957) shows that velocity-dependent forces increase σ_{int} by about 8·5% for the Serber mixture. Donhert and Rojo (1964) find that central velocity-dependent correlations with Serber mixture increase σ_{int} by 14% for nuclear matter. Since the tensor forces reduce the value of σ_{int}, an increase of 8·5% in its value with tensor forces and velocity-dependent correlations seems to be satisfactory.

The experimental value of the harmonic mean energy (Gorbunov and Spiridonov 1958) which is a measure of the resonance peak is $(W_{\mathrm{H}})_{\mathrm{expt}} = \sigma_{\mathrm{int}}/\sigma_{\mathrm{b}} = 39{\cdot}6$ MeV, and our calculated value for the Serber mixture is about 45·3 MeV. Thus, the agreement is satisfactory.

Our calculations further show that the Serber mixture is a more suitable form of interaction.

Finally, we remark that a comparison of our results with those of Goldhammer and Valk (1962) establishes reasonably well the equivalence of hard-core and velocity-dependent potentials in photo-effect calculations for the alpha particle.

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